USB-preprint: SB/F/288-01

Master Canonical Action and BRST Charge of the M Theory Bosonic Five Brane

A. De Castro[1] and A. Restuccia[2†]

Universidad Simón Bolívar, Departamento de Física, AP 89000, Caracas 1080-A, Venezuela

† *Department of Mathematics, Kings College, London*

Abstract

A complete analysis of the canonical structure for a gauge fixed PST bosonic five brane action is performed. This canonical formulation is quadratic in the dependence on the antisymmetric field and it has second class constraints. We remove the second class constraints and a master canonical action with only first class constraints is proposed. The nilpotent BRST charge and its BRST invariant effective theory is constructed. The construction does not assume the existence of the inverse of the induced metric. Singular configurations are then physical ones. We obtain the physical Hamiltonian of the theory and analyze its stability properties. Finally, by studying the algebra of diffeomorphisms we find under mild assumptions the general structure for the Hamiltonian constraint for theories invariant under 6 dimensional diffeomorphisms and we give an algebraic characterization of the constraint associated with the bosonic five brane action. We also identify the constraint for the bosonic five brane action upgraded with a cosmological term, it contains a Born-Infeld type term.

[1]E-mail address: alex@fis.usb.ve

[2]E-mail address: arestu@usb.ve

1 Introduction

In recent years the M5-brane in $D = 11$ has acquired a protagonist role in understanding the duality relations in the M-theory framework. The degrees of freedom of 6d world volume theory of the M5-brane are associated with the super-symmetric tensor multiplet $N = (2, 0)$. This multiplet contains a 2-form gauge field B_{MN} with a self-dual field strength, five scalars and two chiral spinors. The presence of this chiral gauge field was an obstacle to the formulation of a covariant world volume M5-brane action. P. Pasti, D. Sorokin and M.Tonin in 1997 [1] found a way of dealing with this obstacle using the auxiliary scalar field approach, thus allowing the formulation of a covariant super-5-brane action with its usual κ-invariance [2]. However, in order to obtain an operatorial or functional integral quantum formulation of the theory, for example to analyze the quantum stability properties of the theory, it seems necessary to eliminate by partial gauge fixing the auxiliary scalar field of the PST approach, since it is present in the denominator of the Lagrangian. The covariant field equations for the super $M5$-brane were first obtained in [3], [4] and [5] using the superembedding approach.

An action for the M5-brane was independently formulated in a non manifestly covariant way in [6], it corresponds to a gauge fixed version of the covariant proposal. There are two partial gauge fixing conditions that naturally eliminate off the auxiliary scalar field of the PST approach. One of them corresponds to fixing the scalar field as the world volume time and the other as a local world volume spatial coordinate.The latest corresponds to the formulation in [6]. In the first case the action is first-order in time derivatives of the antisymmetric field and in the second one is higher order in time derivatives, moreover it is non polinomial in the dependence of the time derivatives of the antisymmetric field. The first case, although has second class constraints, allows a direct canonical analysis. We remark that the PST formulation as well as the formulation in [6] assume the existence of the inverse of the induced metric.

The canonical Hamiltonian for the 5-brane was first obtained in [7], [8] and also discussed in [9], however in order to analyze the stability properties of the 5-brane and to compare them with the $D = 11$ supermembrane theory it is relevant to go one step further, to include also in the analysis the configurations with zero determinant of the induced metric since as we will show they are physical configurations of the theory, and study then the physical Hamiltonian. That is, the Hamiltonian describing the dynamics of the physical degrees of freedom once the

gauge ones have been eliminated by an admissible gauge fixing procedure.

When the theory is reduced to the subspace of solutions of the field equations for which the induced metric is flat, the Lagrangian of the 5-brane reduces to the one in [10] whose canonical analysis was obtained in [11]. An interesting feature of [11] was that the second class constraints of the original formulation were eliminated giving rise to a canonical Lagrangian with first class constraints only. In this case, its double dimensional reduction yield a canonical formulation allowing two covariant gauge fixing procedures, one of them gives the Hamiltonian formulation of the 4-brane and the other one its dual in terms of the antisymmetric field. The analysis was performed even though the dependence on the time derivatives of the antisymmetric field is non polinomial. The resulting Hamiltonians contain in both cases the typical Born-Infeld structure for the field strength of the gauge vector field and antisymmetric one.

In this work we give first a complete analysis of the canonical structure of the bosonic sector of the 5-brane. We start from the PST action and consider the partial gauge fixing where the scalar field is fixed as the world volume time. The canonical formulation of the $M5$-brane turns out to be quadratic in the dependence on the antisymmetric field, and has second class constraints together with the first class ones which generates the symmetries of the theory. The Hamiltonian in [8] reduces to the one we obtain once their constraints are properly combined. We remove the second class constraints preserving the locality of the field theory and ending up with a master canonical action which only contains first class constraints and is well defined even for singular induced metrics. We refer as singular configurations the ones for which the determinant of the spatial part of the induced metric is zero at some point or neighborhood of the world volume. The algebra of the 6 dimensional diffeomorphisms generated by the first class constraints is explicitly given. It is an open algebra.

This is the first step in the construction of an unconstrained extended phase where a realization of the nilpotent BRST charge may be possible. The BRST charge is a fundamental geometrical object in the formulation of the quantum field theory associated to the canonical Lagrangian. It may also be an important geometrical object in the construction of the Seiberg-Witten map [12] that relates the 5-brane theory formulated in terms of a commutative geometry to a formulation in terms of an associated noncommutative one. The Seiberg-Witten map is a one to one correspondence between gauge equivalent classes, which correspond to BRST equivalence classes in the extended phase space. In particular, this implies that the cohomology

classes of the BRST operators in the commutative realization and in the non commutative one are in one to one correspondence. We construct the nilpotent BRST charge of the theory and its BRST invariant effective theory. The construction involves several steps beyond the standard construction for a closed algebra. In fact, one has to introduce the higher order structure functions of the open algebra. We obtain its physical Hamiltonian and analyze its stability properties remarking that although the Hamiltonian is quadratic on the dependence on the antisymmetric field, its reduction to a flat induced metric gives the Born-Infeld type of Hamiltonian. The improvement is important when the operatorial formulation of the BRST invariant effective action is considered.

Finally, by studying the algebra of diffeomorphisms we find the most general structure for the Hamiltonian constraint in terms of the membrane maps and the antisymmetric field. The corresponding canonical Hamiltonian describe field theories which are invariant under diffeomorphisms over a 6 dimensional world volume with a chiral gauge field. The unique Hamiltonian constraint with quadratic dependence on the antisymmetric field corresponds to the M5-brane. We also identify the constraint associated to the bosonic M5-brane action upgraded with a cosmological term,it involves a Born-Infeld type of term. From the algebraic point of view, the Hamiltonian constraint for the M5-brane is characterized by being the unique one which is polinomial in its dependence on the field strength of the antisymmetric field and is well defined even for singular induced metrics. The BRST invariant action, that we will construct, is characterized also by the same property. Its Lagrangian density is well behaved even for singular configurations of the induced metric. Hence, singular configurations of the metric are allowed as physical configurations. In distinction, the PST covariant action, as well as the one in [6], assume the existence of the inverse of the induced metric at any point of the world volume. In the case of the D=11 Supermembrane, the singular configurations are also physical configurations. This has important consequences with respect to the quantum properties of the theory. In fact, the existence of these configurations together with the supersymmetry render the spectrum continuous from zero to infinite. They are also related to the topology changes on the embedded surface in the target space, without changing the energy of the supermembrane. It is thus very important to take them into account in any analysis of the theory.

Another consequence of the existence of singular configurations is that the static "gauge"

is not a correct gauge fixing of the theory, since giving a singular configuration one cannot gauge transform it to a non singular one (in the static gauge the spatial part of the induced metric is always non singular). If one restricts the theory to the space of nontrivial higher order bundles [13] or equivalent to non trivial wrapping of the 5-brane on the target space, then the singular configurations are avoided and it becomes a correct gauge fixing condition. However the quantum stability analysis of the theory must include all canonical admissible configurations and it is in that case that the static gauge is not allowed. In distinction the light cone gauge is always an admissible gauge. In fact, even the singular configurations can be gauged to it, and of course they remain to be singular.

2 Canonical analysis of the bosonic 5-brane action

We start from the PST action for the M5-brane and consider the gauge in which the scalar field is proportional to the world volume time. This gauge fixing, associated to the gauge symmetry of the auxiliar scalar field, may be implemented directly into the action, since the Fadeev-Popov procedure gives a constant contribution to the measure of the functional integral. We obtain the following Lagrangian density

$$L = 2\sqrt{-\det\left(G_{MN} + G_{M\rho}G_{N\lambda}{}^{\star}H^{\rho\lambda}\sqrt{-G_0}\right)} + \frac{1}{2}{}^{\star}H^{\mu\nu}\partial_0 B_{\mu\nu} + \frac{1}{4}\epsilon_{\mu\nu\rho\lambda\sigma}\frac{G^{0\rho}}{G^{00}}{}^{\star}H^{\mu\nu\star}H^{\lambda\sigma} \tag{1}$$

Where B denotes the antisymmetric gauge field and H is the self-dual field strength $H = dB$. The 6 dimensional world volume indices are denoted by $M, N = 0, 1, \cdots, 5$ while the spatial ones by $\mu\nu = 1, \cdots, 5$. G_{MN} in terms of the 5-brane maps X^a is given by the induced expression $G_{MN} = \partial_M X^a \partial_N X_a$, where a denotes the $D = 11$ Minkowski indices. In the spatial 5-dimensional world volume we denote:

$$H_{\rho\lambda\sigma} = \partial_\rho B_{\lambda\sigma} + \partial_\sigma B_{\rho\lambda} + \partial_\lambda B_{\sigma\rho} \tag{2}$$

$${}^{\star}H^{\mu\nu} \equiv \frac{1}{6}\epsilon^{\mu\nu\gamma\delta\lambda}H_{\gamma\delta\lambda} \tag{3}$$

Where ${}^\star H^{\mu\nu}$ is a contravariant density. In order to analyze the Hamiltonian structure of this action it is convenient to introduce the **ADM** parametrization of the metric

$$
\begin{aligned}
G_{00} &\equiv -(n^2 - N_\lambda N^\lambda) = \dot{X}^a \dot{X}_a \\
G_{0\mu} &\equiv N_\mu = \dot{X}^a \partial_\mu X_a \\
G_{\mu\nu} &\equiv g_{\mu\nu} = \partial_\mu X^a \partial_\nu X_a \\
N^\mu &\equiv g^{\mu\nu} N_\nu, \\
G^{00} &= -(\frac{1}{n^2}), \\
G^{0\mu} &= \frac{N^\mu}{n^2}, \\
G^{\mu\nu} &= g^{\mu\nu} - \frac{N^\mu N^\nu}{n^2}, \\
\det(G_{MN}) &= -n^2 \det(g_{\mu\nu}).
\end{aligned} \tag{4}
$$

They allow us to rewrite (1) in the form

$$
L = 2n\sqrt{gM} - \frac{1}{4} N^\rho \hat{V}_\rho + \frac{1}{2} {}^\star H^{\mu\nu} \partial_0 B_{\mu\nu} \tag{5}
$$

where

$$
\begin{aligned}
g &= \det g_{\mu\nu} \\
M &\equiv 1 + \hat{y} + \hat{z} \\
\hat{y} &\equiv \frac{1}{2} g^{-1} {}^\star H^{\alpha\beta} {}^\star H_{\alpha\beta} \\
\hat{z} &\equiv \frac{1}{64} g^{-1} g^{\mu\nu} \hat{V}_\mu \hat{V}_\nu \\
\hat{V}_\mu &= \epsilon_{\mu\alpha\beta\gamma\delta} {}^\star H^{\alpha\beta} {}^\star H^{\gamma\delta}
\end{aligned} \tag{6}
$$

The spatial world volume indices are raised and lowered with the induced metric, the need of using the inverse of $g_{\mu\nu}$ will be later on relaxed. The term gM is precisely

$$
\det(g_{\mu\nu} + g^{-1/2} {}^\star H_{\mu\nu}) = gM \tag{7}
$$

The conjugate momenta to X^a may be directly evaluated. It is

$$\Pi_a = 2\frac{g^{1/2}}{n}(-\dot{X}_a + N^\lambda \partial_\lambda X_a)M^{1/2} - \frac{1}{4}\hat{V}^\rho \partial_\rho X_a \tag{8}$$

We can guess that the diffeomorphisms constraints have a similar diffeomorphisms constraints structure as in string and membrane theory. We finally obtain

$$\begin{aligned} \hat{\phi} &= \frac{1}{2}\Pi^2 + g(2\hat{y} + 2) = 0 & (9) \\ \hat{\phi}_\alpha &= \Pi_a \partial_\alpha X^a + \frac{1}{4}\hat{V}_\alpha = 0 & (10) \end{aligned}$$

The conjugate momenta to $B_{\mu\nu}$ will be denoted $P^{\mu\nu}$ and satisfies the constraint

$$\Omega^{\mu\nu} = P^{\mu\nu} - {}^\star H^{\mu\nu} = 0 \tag{11}$$

The standard canonical analysis shows that (9), (10) and (11) are the only constraints of the theory and they are a mixture of first and second class constraints. The canonical Hamiltonian is a linear combination of these constraints

$$\mathcal{H}_c = \Lambda\hat{\phi} + \Lambda^\alpha \hat{\phi}_\alpha + \Lambda_{\mu\nu}\Omega^{\mu\nu} \tag{12}$$

We will now consider a more general canonical formulation with first class constraints only, which under partial gauge fixing reduces to (12). Such a formulation can be obtained by several approaches. Following [14], we deduce from (11)the first class constraints for the antisymmetric field

$$\begin{aligned} \Omega^{5i} &= P^{5i} - {}^\star H^{5i} = 0 & (13) \\ \Omega^j &= \partial_\mu P^{\mu j} = 0, \quad i = 1, 2, 3, 4. & (14) \end{aligned}$$

The diffeomorphisms constraints must be modified in order to obtain a complete set of first class constraints and we propose the following structure:

$$\phi = \frac{1}{2}\Pi^2 + g(2y+2) = 0 \tag{15}$$

$$\phi_\alpha = \Pi_a \partial_\alpha X^a + \frac{1}{4} V_\alpha = 0 \tag{16}$$

where

$$y = \frac{1}{8} g^{-1} (P^{\alpha\beta} + {}^\star H^{\alpha\beta})(P_{\alpha\beta} + {}^\star H_{\alpha\beta}) \tag{17}$$

$$V_\mu = \frac{1}{4} \epsilon_{\mu\alpha\beta\gamma\delta} (P^{\alpha\beta} + {}^\star H^{\alpha\beta})(P^{\gamma\delta} + {}^\star H^{\gamma\delta}) \tag{18}$$

Ω^{5i} and Ω^i commute between themselves and with ϕ and ϕ_α. The simplectic space of the antisymmetric field and its conjugate momenta has been decomposed in terms of the local commuting coordinates

$$P^{\mu\nu} + {}^\star H^{\mu\nu}$$

and

$$P^{\mu\nu} - {}^\star H^{\mu\nu}$$

with Poisson brackets

$$\{P^{\mu\nu} + {}^\star H^{\mu\nu}, P^{\alpha\beta} - {}^\star H^{\alpha\beta}\} = 0 \tag{19}$$

the latest coordinate may be fixed by a gauge condition associated to constraints (13) and (14). The procedure to obtain (13), (14), (15) and (16) is deductive, however we have only presented the resulting expressions whose required properties may be directly checked [14]. ϕ and ϕ_α generates the algebra of diffeomorphism on the world volume: The canonical Hamiltonian is then the linear combination of the first class constraints. We have broken even further the manifest covariance of the formulation, nevertheless we have now a polinomic formulation of the theory in terms of first class constraints only. We remark that the square root characteristic

of the Born-Infeld action for the 5-brane has now disappear. These properties ensure the construction of a polinomic physical Hamiltonian and a polinomic BRST invariant formulation of the theory. We will consider both points in the following sections.

The algebra generated by the first class constraints (13), (14), (15) and (16) is given by:

$$\begin{aligned} \{\phi_\rho, \phi'_\mu\} &= \phi_\mu \partial_\rho \delta + \phi'_\rho \partial_\mu \delta - (\partial_\gamma P^{\gamma\delta}) l^{\alpha\beta} \epsilon_{\mu\delta\rho\alpha\beta} \cdot \delta && (20) \\ \{\phi_\rho, \phi'\} &= (\phi + \phi')\partial_\rho \delta - 4(\partial_\lambda P^{\lambda\alpha}) l_{\alpha\rho} \cdot \delta && (21) \\ \{\phi, \phi'\} &= (\mathcal{C}^{\rho\sigma}\phi_\rho + \mathcal{C}'^{\rho\sigma}\phi'_\rho)\partial_\sigma \delta && (22) \end{aligned}$$

where

$$\mathcal{C}^{\sigma\lambda} = 4(g g^{\sigma\lambda} + g_{\alpha\beta} l^{\alpha\sigma} l^{\beta\lambda}), \quad l^{\alpha\beta} \equiv \frac{1}{2}(P^{\alpha\beta} + {}^\star H^{\alpha\beta}) \qquad (23)$$

We remark that the first class constraints, as well as the structure functions of the algebra, depend only on $g_{\mu\nu}$ and not on its inverse. This is an important difference with respect to the covariant formulation of the theory, where the metric is assumed to have an inverse. It is well known in the case of the $D = 11$ supermembrane that the singular configurations, which in this theory are string like configurations, are responsible for the continuous spectrum of the Hamiltonian as well as for topological changes of the brane. [15], [16] [17].

3 BRST effective action

We have succeeded in finding a formulation of the 5-brane which is polinomic on the fields and with first class constraints only. We will now construct the associated nilpotent BRST charge and the BRST invariant effective action. The BRST charge is the fundamental geometrical object in the quantum analysis of the theory and in the construction of the map relating the 5-brane formulation to a non commutative geometry.

The effective action will depend on the covariant induced metric only, and not in its inverse, so is well behaved even on singular configurations where the determinant of the induced metric becomes zero.

In order to have a realization of the generators of spatial diffeomorphisms ϕ_ρ, in the whole space of geometrical objects including the antisymmetric field and its conjugate momenta we will consider a reduction of the phase space by restricting to configurations satisfying

$$\partial_\mu P^{\mu\nu} = 0 \tag{24}$$

We may performed this restriction starting with the whole extended phase space and imposing finally at the level of the effective action a canonical gauge fixing condition associated to (24):

$$\chi(B, P) = 0 \tag{25}$$

depending only on B and P fields.

Otherwise we may start restricting the phase space by conditions (24) and (25) and working out the nilpotent BRST charge on the subspace of phase space. In our case, the latest approach becomes slightly more direct. This is so because $\partial_\mu P^{\mu\nu}$ commutes with all constraints of the theory and with all phase space coordinates except $B_{\mu\nu}$. However $B_{\mu\nu}$ appears in the constraints only through ${}^\star H^{\alpha\beta}$, which commutes with $\partial_\mu P^{\mu\nu}$. Consequently, in the BRST construction we may work directly with Poisson brackets even so we are restricted by (24) and (25) (Dirac brackets are the same as Poisson brackets). We will follow this latest approach, for simplicity we require

$$\{\chi, \chi'\} = 0 \tag{26}$$

The algebra of diffeomorphism obtained in section §2 is an open algebra, that is, the structure functions depend on the fields. Consequently, the construction of the nilpotent BRST charges requires several additional steps beyond the standard construction for a closed algebra[18]. They involve the higher order structure functions of the algebra. We introduce the ghosts C and C^ρ associated to ϕ and $\phi_\rho$and its conjugate momenta μ and μ_ρ respectively. We start from the Poisson brackets

$$\{\phi, \phi'\} = (\mathcal{C}^{\sigma\lambda}\phi_\sigma + \mathcal{C}'^{\sigma\lambda}\phi'^\sigma)\partial_\lambda\delta \tag{27}$$

where

$$\mathcal{C}^{\sigma\lambda} = 4(gg^{\sigma\lambda} + g_{\alpha\beta}l^{\alpha\sigma}l^{\beta\lambda}) \tag{28}$$

It is convenient to extend the other generator of diffeomorphisms ϕ_ρ from the beginning in order to simplify the construction. We define

$$\widetilde{\phi}_\rho = \phi_\rho + 2\mu\partial_\rho C + \partial_\rho\mu \cdot C + \mu_\lambda\partial_\rho C^\lambda + \partial_\lambda(\mu_\rho C^\lambda) \tag{29}$$

We then have

$$\begin{aligned}
\{\langle\xi^\rho\widetilde{\phi}_\rho\rangle, C\} &= -2\xi^\rho\partial_\rho C + \partial_\rho(\xi^\rho C) = -\xi^\rho\partial_\rho C + \partial_\rho\xi^\rho C, && (30)\\
\{\langle\xi^\rho\widetilde{\phi}_\rho\rangle, \mu\} &= -\xi^\rho\partial_\rho\mu - 2\partial_\rho\xi^\rho\mu && (31)
\end{aligned}$$

with the right density weights to obtain

$$\{\langle\xi^\rho\widetilde{\phi}_\rho\rangle, \mu C\} = -\xi^\rho\partial_\rho(\mu C) - \partial_\rho\xi^\rho(\mu C), \tag{32}$$

Consequently, we have

$$\{\langle\xi^\rho\widetilde{\phi}_\rho\rangle, \langle C\phi\rangle\} = \langle -\partial_\rho(\xi^\rho C\phi)\rangle = 0 \tag{33}$$

We will drops all boundary terms, that is we will assume a closed compact spatial world volume. We also get

$$\{\langle\xi^\rho\widetilde{\phi}_\rho\rangle, C^\mu\} = -\xi^\rho\partial_\rho C^\mu + \partial_\rho\xi^\mu C^\rho, \tag{34}$$

since C^μ transforms as a contravariant vector
The introduction of $\widetilde{\phi}_\rho$ simplifies the construction since its action on any new term in the BRST charge reduces only to count weights for objects which transform under diffeomorphisms as densities.

We may start considering

$$Q = \langle C\phi + C^{\rho}\widetilde{\phi}_{\rho} - \mu_{\sigma} C \partial_{\lambda} C \mathcal{C}^{\sigma\lambda} - C^{\rho}\mu_{\lambda}\partial_{\rho}C^{\lambda}\rangle + Q^{H} + Q^{A} \equiv Q^{C} + Q^{H} + Q^{A} \tag{35}$$

Where Q^H denotes the terms involving the higher order structure functions of the diffeomorphism algebra. Q^A denotes the contributions to Q of the constraints associated to the antisymmetric field. It commutes with $Q^C + Q^H$, hence we will consider it after the complete evaluation of Q^C and Q^H. We will systematically add the higher order terms in derivatives of C in order to ensure the nilpotency of Q.
We get

$$\{\langle C\phi\rangle, \langle C\phi\rangle\} = \langle 2C\partial_{\lambda} C \mathcal{C}^{\rho\lambda}\phi_{\rho}\rangle \tag{36}$$

$$2\{\langle C^{\rho}\widetilde{\phi}_{\rho}\rangle, \langle -\mu_{\sigma} C\partial_{\lambda} C\mathcal{C}^{\sigma\lambda}\rangle\} = -\langle 2C\partial_{\lambda}C\mathcal{C}^{\rho\lambda}\phi_{\rho}\rangle - 2\langle [\mu_{\hat{\lambda}}\partial_{\rho}C^{\hat{\lambda}} + \partial_{\hat{\lambda}}(\mu_{\rho}C^{\hat{\lambda}})]C\partial_{\lambda}C\mathcal{C}^{\rho\lambda}\rangle \tag{37}$$

$$2\{\langle -\mu_{\sigma}C\partial_{\lambda}C\mathcal{C}^{\sigma\lambda}\rangle, \langle -C^{\rho}\mu_{\hat{\lambda}}\partial_{\rho}C^{\hat{\lambda}}\rangle\} = 2\langle C\partial_{\lambda}C\mathcal{C}^{\sigma\lambda}\mu_{\nu}\partial_{\sigma}C^{\nu}\rangle - 2\langle C\partial_{\lambda}C\mathcal{C}^{\sigma\lambda}\partial_{\rho}(C^{\rho}\mu_{\sigma})\rangle \tag{38}$$

$$2\{\langle C^{\rho}\widetilde{\phi}_{\rho}\rangle, \langle C^{\lambda}\widetilde{\phi}_{\lambda}\rangle\} = -2C^{\nu}\partial_{\nu}C^{\rho}\widetilde{\phi}_{\rho} \tag{39}$$

$$2\{\langle C^{\rho}\widetilde{\phi}_{\rho}\rangle, \langle -C^{\nu}\mu_{\lambda}\partial_{\nu}C^{\lambda}\rangle\} = 2C^{\nu}\partial_{\nu}C^{\rho}\widetilde{\phi}_{\rho} \tag{40}$$

$$2\{\langle C^{\hat{\nu}}\mu_{\hat{\lambda}}\partial_{\hat{\nu}}C^{\hat{\lambda}}\rangle, \langle C^{\nu}\mu_{\lambda}\partial_{\nu}C^{\lambda}\rangle\} = 0 \tag{41}$$

All these brackets cancel between each other, which is the standard situation for a closed algebra. However, since $\mathcal{C}^{\sigma\lambda}$ are field dependent, we have additional contributions to the evaluation of the $\{Q, Q\}$.

We obtain

$$2\{\langle C\phi\rangle, \langle -\mu_\sigma C\partial_\lambda C\mathcal{C}^{\sigma\lambda}\rangle\} = \langle 2\mathcal{C}_1^{\mu\nu,\sigma\lambda}\mu_\sigma C\partial_\lambda C\partial_\mu C\phi_\nu\rangle \tag{42}$$

where

$$\mathcal{C}_1^{\mu\nu,\sigma\lambda} = 4(-2gg^{\sigma\lambda}g^{\mu\nu} + gg^{\sigma\mu}g^{\nu\lambda} + gg^{\sigma\nu}g^{\mu\lambda} - l^{\mu\sigma}l^{\nu\lambda} - l^{\nu\sigma}l^{\mu\lambda}) \tag{43}$$

The first contribution to Q^H in then

$$Q^H = \langle -\frac{1}{2}\mathcal{C}_1^{\mu\nu,\sigma\lambda}\mu_\nu\mu_\sigma C\partial_\lambda C\partial_\mu C + \cdots\rangle \tag{44}$$

Notice that the commutator

$$\{\langle -C^\rho\mu_\beta\partial_\rho C^\beta\rangle, -\frac{1}{2}\mathcal{C}_1^{\mu\nu,\sigma\lambda}\mu_\nu\mu_\sigma C\partial_\lambda C\partial_\mu C\}, \tag{45}$$

gives a contribution which combines with (42) to give

$$\langle 2\mathcal{C}_1^{\mu\nu,\sigma\lambda}\mu_\sigma C\partial_\lambda C\partial_\mu C\widetilde{\phi}_\nu\rangle \tag{46}$$

which is canceled by the commutator

$$2\{\langle C^\rho\widetilde{\phi}_\rho\rangle, -\frac{1}{2}\mathcal{C}_1^{\mu\nu,\sigma\lambda}\mu_\nu\mu_\sigma\partial_\lambda C\partial_\mu CC\}, \tag{47}$$

$\mathcal{C}_1^{\mu\nu,\sigma\lambda}$ is again field dependent so there are further contributions from the commutator with $\langle C\phi\rangle$.

The next contribution is from the commutators

$$2\{\langle C\phi\rangle, \langle -\frac{1}{2}\mathcal{C}_1^{\mu\nu,\sigma\lambda}\mu_\nu\mu_\sigma\partial_\lambda C\partial_\mu CC\rangle\} + \{\langle -\mu_\sigma C\partial_\lambda C\mathcal{C}^{\sigma\lambda}\rangle, \langle -\mu_\sigma C\partial_\lambda C\mathcal{C}^{\sigma\lambda}\rangle\} \tag{48}$$

which require the addition of a term

$$\langle 8g(\mu^\lambda\partial_\lambda C)^3 C\rangle \tag{49}$$

to Q.

The commutator of $\langle C^\rho\widetilde{\phi}_\rho + \mu_\lambda C^\rho\partial_\rho C^\lambda\rangle$ with (49) cancels the commutators in (48). Furthermore, the following commutators

$$2\{\langle C\phi\rangle, \langle 8g(\mu^\lambda\partial_\lambda C)^3 C\rangle\} + 2\{\langle -\mu_\sigma C\partial_\lambda C\mathcal{C}^{\sigma\lambda}\rangle, \langle -\frac{1}{2}\mathcal{C}_1^{\mu\nu,\sigma\lambda}\mu_\nu\mu_\sigma C\partial_\lambda C\partial_\mu C\rangle\} \tag{50}$$

require a higher order term in Q. It is

$$\langle 10g(\mu^\lambda\partial_\lambda C)^4 C\rangle \tag{51}$$

The commutator of $\langle C^\rho\widetilde{\phi}_\rho + \mu_\lambda C^\rho\partial_\rho C^\lambda\rangle$ with (51) cancels the commutators in (50). Finally the commutators

$$2\{\langle C\phi\rangle, \langle 10g(\mu^\lambda\partial_\lambda C)^4 C\rangle\} + \{\langle -\frac{1}{2}\mathcal{C}_1^{\mu\nu,\sigma\lambda}\mu_\nu\mu_\sigma C\partial_\lambda C\partial_\mu C\rangle, \langle -\frac{1}{2}\mathcal{C}_1^{\mu\nu,\sigma\lambda}\mu_\nu\mu_\sigma C\partial_\lambda C\partial_\mu C\rangle\} \tag{52}$$

require a final new terms in Q:

$$\{\langle 12g(\mu^\lambda\partial_\lambda C)^5 C\rangle\} \tag{53}$$

The commutator of $\langle C^\rho\widetilde{\phi}_\rho\partial_\rho C^\lambda\rangle$ with (53) cancels the commutators in (52). Finally the commutator of $\langle C\phi\rangle$ with (53) gives zero since the order in derivatives of C is 6. The nilpotency of Q has then been obtained.

The final form of the BRST charge is thus given by

$$\begin{aligned} Q &= \langle C\phi + C^\rho\widetilde{\phi}_\rho + \mathcal{C}^{\sigma\lambda}\mu_\sigma\partial_\lambda CC + \mu_\lambda C^\rho\partial_\rho C^\lambda \\ &- \frac{1}{2}\mathcal{C}_1^{\mu\nu,\sigma\lambda}\mu_\sigma\partial_\lambda C\mu_\nu\partial_\mu CC + 8g(\mu^\lambda\partial_\lambda C)^3 C \end{aligned}$$

$$+ \quad 10g(\mu^\lambda\partial_\lambda C)^4 C + 12g(\mu^\lambda\partial_\lambda C)^5 C + \widehat{C}\partial_i\hat{\mu}^i + \hat{C}_i(P^{5i} - {}^\star H^{5i})\rangle \tag{54}$$

Where we have also included Q^A, the contributions of the first class constraints associated to the antisymmetric field, which under assumption (24) become a reducible set.

We may now construct the BRST invariant effective action. We follow the BFV approach [19] [20], and we obtain

$$\begin{aligned} S_{eff} &= \int d^5\sigma d\tau \Big[\mu\dot{C} + \mu_\rho\dot{C}^\rho + P^{\mu\nu}\dot{B}_{\mu\nu} + \Pi_a\dot{X}^a + \hat{\mu}^i\dot{\hat{C}}_i + \widehat{\mu}\dot{\widehat{C}} + \hat{\delta}(\lambda\mu + \lambda^\rho\mu_\rho + \hat{\lambda}_i\hat{\mu}^i + \hat{\lambda}\hat{\mu}) \\ &+ \hat{\delta}(\bar{C}\chi + \bar{C}_\rho\chi^\rho + \bar{\hat{C}}^i\hat{\chi}_i + \bar{\hat{C}}_1\chi_1 + \bar{C}_2\chi_2 + \bar{C}_3\chi_3 + \bar{C}_4\chi_4)\Big] \end{aligned} \tag{55}$$

Where λ, λ_ρ are the Lagrange multipliers associated to the diffeomorphisms constraints while $\hat{\lambda}_i$ is associated to the constraint on the antisymmetric field. χ, χ^ρ and $\hat{\chi}_i$ are the associated gauge fixing functions. χ_1, χ_2, χ_3 and χ_4 are the gauge fixing associated to the reducibility of the constraints on the antisymmetric field. We take the following gauge fixing functions:

$$\begin{aligned} \chi &= \lambda - \frac{1}{\sqrt{W}} \\ \chi^\rho &= \lambda^\rho \\ \hat{\chi}_i &= \hat{\lambda}_i \end{aligned} \tag{56}$$

Where we have introduced a non singular metric over the spatial world volume. This is so, because λ transforms as a density under diffeomorphisms. In some particular cases we may take $W = 1$. The introduction of this metric occurs in a similar way as for the $D = 11$ supermembrane, through the gauge fixing procedure.

χ_1, χ_2, χ_3 and χ_4 fix the longitudinal parts, with respect to the covariant derivative constructed with the metric that has been introduced, of $\hat{C}^i$, $\bar{\hat{C}}_i$, $\hat{B}_i$ and $\hat{\lambda}_i$, where $\hat{\delta}\bar{\hat{C}}_i = \hat{B}_i$, $\hat{\delta}$ denotes the BRST transformation. We refer to [20] for details of the construction.

We may now integrate out, or eliminate from the field equations, the auxiliary field. We obtain finally:

$$S_{eff} = \int d^5\sigma d\tau[\mu\dot{C} + \mu_\rho\dot{C}^\rho + P^{\mu\nu}\dot{B}_{\mu\nu} + \Pi_a\dot{X}^a + \hat{\mu}^i\dot{\hat{C}}_i + \widehat{\mu}\dot{\widehat{C}} + \frac{1}{\sqrt{W}}\hat{\delta}\mu] \tag{57}$$

where $\hat{\delta}\mu$ is the BRST transformed of μ.
We notice that in the construction

$$\{Q, Q'\}_{\mathcal{D}} = \{Q, Q'\} = 0 \tag{58}$$

Where the first bracket is a Dirac bracket, while the second one is a Poisson bracket. Moreover the Dirac bracket of Q with any coordinate of the phase space, excluding $B_{\mu\nu}$, is the same as its Poisson bracket. However the Dirac bracket of Q with $\langle P^{\mu\nu}\dot{B}_{\mu\nu}\rangle$ is the same as its Poisson bracket, in the subspace (24). Consequently the effective action is BRST invariant, since its kinetic term transform as

$$\hat{\delta}S_{eff} = \int d^5\sigma d\tau \hat{\delta}[\mu\dot{C} + \mu_\rho \dot{C}^\rho + P^{\mu\nu}\dot{B}_{\mu\nu} + \Pi_a \dot{X}^a + \hat{\mu}^i \dot{\hat{C}}_i + \widehat{\mu}\dot{\widehat{C}}] = \int d\tau \dot{Q} = 0 \tag{59}$$

provided initial and final conditions on the ghost fields are imposed [20] [21], as usual.
We notice that

$$\begin{aligned}\langle \frac{1}{\sqrt{W}}\hat{\delta}\mu\rangle &= \langle \frac{1}{\sqrt{W}}(\phi + C^{\sigma\lambda}\mu_\sigma\partial_\lambda C - \frac{1}{2}C_1^{\mu\nu,\sigma\lambda}\mu_\sigma\partial_\lambda C\mu_\nu\partial_\mu C + 8g(\mu^\lambda\partial_\lambda C)^3 \\ &+ 10g(\mu^\lambda\partial_\lambda C)^4 + 12g(\mu^\lambda\partial_\lambda C)^5)\rangle \end{aligned} \tag{60}$$

is not only manifestly BRST invariant, but also well behaved even on singular configurations of the induced metric, where its determinant is zero, in spite of the fact that $\mu^\lambda = g^{\lambda\sigma}\mu_\sigma$. In fact all these term may be rewritten in terms of the totally antisymmetric symbol $\epsilon^{\alpha\beta\mu\nu\lambda}$ and the covariant metric $g_{\mu\nu}$ only.

4 Light Cone Gauge Hamiltonian of the M5-brane

In the previous section we obtained the general formulation of the BRST invariant effective action of the M5-brane in the covariant gauge (56), we showed that the action may be expressed in a manifestly Lorentz covariant way (in the target space), without restricting the induced metric to have an inverse. In this section we will analyze some stability properties of the

effective action. To do so we will start from (55) and fix the light cone gauge, which leads to physical Hamiltonian after elimination of the constraints.
Consider the light cone gauge fixing conditions:

$$
\begin{aligned}
X^{+} &= \Pi_0^{+}\tau & (61)\\
\Pi^{+} &= \Pi_0^{+}\sqrt{W}, & (62)
\end{aligned}
$$

We will later on discuss the gauge fixing conditions for the antisymmetric field.
The LCG allows to reduce canonically the phase space to its transverse part only. This is achieved by solving explicitly the constraint (15) for Π_+, X^- is eliminated from the constraint (16) provided an integrability condition is satisfy. This condition is a first class constraint which generates the volume preserving diffeomorphisms.
The canonical reduction of the effective action yields after the elimination of the ghost, antighost fields and Lagrange multipliers to the canonical Lagrangian:

$$\widetilde{L} = \Pi_M \dot{X}^M + P_{\mu\nu}\dot{B}^{\mu\nu} - \mathcal{H}_p \tag{63}$$

where

$$\mathcal{H}_p = \frac{1}{2}\Pi^M\Pi_M + g2(y+1) + \Theta_{5i}\Omega^{5i} + \Theta_j\Omega^j + \Lambda^{\alpha\beta}\Omega_{\alpha\beta} \tag{64}$$

M denotes the light cone transverse coordinates $M = 1, \cdots, 9$. The explicit expression for y is given in (17). $\Lambda^{\alpha\beta}$ is the antisymmetric Lagrange multiplier associated to the volume preserving diffeomorphism generated by $\Omega_{\alpha\beta}$. The explicit expression for $\Omega_{\alpha\beta}$ is

$$\Omega_{\alpha\beta} = \partial_\beta\{\frac{1}{\Pi_0^+\sqrt{W}}[\Pi_M\partial_\alpha X^M + \frac{1}{4}V_\alpha]\} - \partial_\alpha\{\frac{1}{\Pi_0^+\sqrt{W}}[\Pi_M\partial_\beta X^M + \frac{1}{4}V_\beta]\}. \tag{65}$$

$\Omega_{\alpha\beta} = 0$ is the local integrability condition in order to eliminate X^- from (16). There is also a global integrability condition given by

$$\oint_c \frac{1}{\Pi_0^+\sqrt{W}}[\Pi_M\partial_\alpha X^M + \frac{1}{4}V_\alpha]d\sigma^\alpha = 0 \tag{66}$$

which ensures that X^- is a uniform scalar over the spatial world volume. C is a basis of homology of dimension one. If a compactified target space is assumed, the right hand side of (66) may be proportional to an integer.

We have not fix the gauge associated to the antisymmetric field. There are at least three interesting partial gauge fixing conditions in this respect:

- The gauge fixing yielding

$$P^{\mu\nu} = {}^{\star}H^{\mu\nu}$$

  as used in the formulation in [6]

- The gauge condition

$$B_{5i} = 0$$

  which after double dimensional reduction of the theory yields the formulation of 4-brane in terms of the antisymmetric field [11]

- The gauge

$$B^T_{\mu\nu} = 0$$

  where T denotes transverse with respect to the derivatives operation. It yields after double dimensional reduction to the formulation of the 4-brane in terms of the Born-Infeld $U(1)$ vector field [11].

The absolute minimum of the Hamiltonian is obtained at the configurations satisfying

$$\begin{aligned} \Pi_a &= 0, \\ g &= 0, \\ l^{\mu\nu} l^{\alpha\beta} g_{\mu\alpha} g_{\nu\beta} &= 0, \end{aligned} \tag{67}$$

over any point of the spatial world volume Σ_5, which we assume is closed without boundary.

If $l^{\mu\nu}=0$, then the set Ω in the space of physical configurations at which the minimum is obtained, becomes the set of maps X^a from Σ_5 to the target space, depending on four linear combinations of the local coordinates. That is, all maps X^a are functions of at most four of them. It is an infinite dimensional space of $1,2,3$ and 4 branes. The degeneracy of Ω is analogous to the one that occurs for the $D=11$ supermembrane. There are string like spikes in that case, which are responsible together with supersymmetry for the continuous spectrum of the supermembrane. The degeneracy of the world volume may be pictured as lower p-branes emerging form the world volume which may have free ends on not. It can happen that the other end is plugged into another disconnected sector of the world volume. Such configuration is physically equivalent to the disconnected one, because the tubes not carry any energy. There is then, instability even in the topology of the membrane [17]. These are general features of all p-brane which are also valid for the 5-brane, in spite of the fact that the known covariant formulation imposes restrictions to those singular configurations. The next question that one may ask is if these instabilities go away if the antisymmetric field is of maximum rank, using the fact that it enters in the action as a quadratic form. That is, if we consider the antisymmetric field living on a nontrivial higher order bundle (thus avoiding the $l^{\mu\nu}=0$ configuration), can we still have degenerate configurations?. The answer to this question is that even if we assume the antisymmetric field of maximum rank at any point of Σ_5, there may be string like spikes with zero energy emerging from the world volume. This is so because the antisymmetric field $l^{\mu\nu}$ at any point of Σ_5, has always at least one zero eigenvalue. Its eigenvector being the topological vector V_μ, which is independent of the induced metric. In fact

$$l^{\mu\nu}V_\nu=0 \tag{68}$$

at any point of Σ_5.

If V_μ is a zero vector, then $l^{\mu\nu}$ has three eigenvectors with zero eigenvalue, and the following argument will be also valid.

We consider the following string like configurations

$$X^a \;=\; X^a(Y),$$

$$\partial_\mu Y = \phi V_\mu, \tag{69}$$

where ϕ is scalar field over Σ_5 which will be determined.

We may take the family of curves over Σ_5 tangent, at any point of an open neighborhood, to V_μ. We assume $V_\mu \neq 0$ on that open set. We then choose the σ^5 coordinate along these curves. We then have:

$$\begin{aligned} \partial_i Y &= 0, \\ \partial_5 Y &= \phi V_5, \end{aligned} \tag{70}$$

The solution to this equation is given by

$$\phi = \frac{f(\sigma^5)}{V_5}, \quad \partial_5 Y = f(\sigma^5) \tag{71}$$

Where f is an arbitrary scalar field over Σ_5.

We then conclude, that the quadratic term in the Hamiltonian arising from the antisymmetric field is zero for any of these string like configurations. That is, even when $l^{\mu\nu}$ is of maximum rank at any point of Σ_5 the world volume can degenerate to acquire string like spikes with zero energy. We notice that when $\Pi_a = 0$, as we are assuming, the admissible configurations for the antisymmetric field are restricted by

$$\partial_\mu X^- = -\frac{1}{4} V_\mu, \tag{72}$$

even so there are admissible configurations with $V_\mu \neq 0$ and of course with $V_\mu = 0$. In the latest case we replace in the above argument V_μ by one of the eigenvectors of $l^{\mu\nu}$ with zero eigenvalue.

Finally, we discuss topological conditions which prevent (classically) the existence of degenerated spikes in the world volume. The condition we are going to discuss is not related to a BPS bound in the case of the 5-brane.

The integral of the determinant of the induced metric may be expressed as:

$$\int_{\Sigma_5} \left(dX^a \wedge dX^b \wedge dX^c \wedge dX^d \wedge dX^e\right)^* \left(\frac{dX^a \wedge \cdots \wedge dX^e}{\sqrt{W}}\right) \tag{73}$$

Where $*$ denotes the Hodge dual. A nontrivial minimum of this expression is achieved when

$$ {}^{*}U \equiv * \left(dX^a \wedge \cdots \wedge dX^b\right) = \text{constant} \cdot \text{integer} \tag{74} $$

for a set of five maps, $\hat{X}^1, \cdots, \hat{X}^5$. U is then the curvature of a potential 4-form living on a non trivial higher order bundle [13], [22]. In particular (73) implies that $dX^1, \cdots, dX^5$ are closed, non exact, 1-forms.

We may now show that the set Ω of configurations at which this local minimum is obtained is finite dimensional. We consider any other set of maps $X^1, \cdots, X^5$ on the same higher order bundle. We have

$$ dX^1 \wedge \cdots \wedge dX^5 = U + \delta U \tag{75} $$

where δU is exact. We then obtain

$$ \int_{\Sigma_5} (U + \delta U) * (U + \delta U) = \int_{\Sigma_5} U * U + \int_{\Sigma_5} \delta U * \delta U \geq \int_{\Sigma_5} U * U \tag{76} $$

The equality is obtained when $\delta U = 0$. The question is then if we can perform a deformation of $\hat{X}^1, \cdots, \hat{X}^5$ such that U is preserved. This is always possible since we may consider

$$ \delta X^5 = f(\hat{X}^1, \cdots, \hat{X}^4) \tag{77} $$

where f is an arbitrary scalar over Σ_5. It seems then that the same infinite dimensional space of configurations Ω will still exists. However, under assumption (74), they can be removed by a volume preserving diffeomorphism. In fact, if U is preserved under the deformation, $\delta U = 0$, then the deformed potential must satisfy

$$ \begin{aligned} U &= dV \\ \delta V &= d\Lambda \end{aligned} \tag{78} $$

However, under volume preserving diffeomorphisms with group parameter

$$\xi^{\mu} = \epsilon^{\mu\nu\lambda\rho\sigma}\partial_{\nu}\lambda_{\lambda\rho\sigma} \tag{79}$$

we obtain

$$\delta\left(\hat{X}^{a}d\hat{X}^{b}\wedge d\hat{X}^{c}\wedge d\hat{X}^{d}\wedge d\hat{X}^{e}\epsilon_{abcde}\right) = d({}^{*}U\lambda), \tag{80}$$

${}^{*}U$ being constant by (74). We may then cancel the deformation and remove the degeneracy of Ω arising from exact forms in (78). We are thus left with the cohomology classes of closed forms only. The functional space Ω of minimal configurations is then finite dimensional.

5 Algebra of diffeomorphisms on a 6 dimensional world volume

We will find in this section all possible field theories, realized in terms of the fields X^a and $B_{\mu\nu}$ which are invariant under 6 dimensional diffeomorphisms. To do so, we will find the general structure of the Hamiltonian constraint by imposing the closure of the algebra of diffeomorphism. It turns out that the only polinomic constraint which depends on $g_{\mu\nu}$ and not on its inverse is the associated to the 5-brane theory. The algebra is the following

$$\begin{aligned}
\{\phi_\rho, \phi'_\mu\} &= \phi_\mu\partial_\rho\delta + \phi'_\rho\partial_\mu\delta - (\partial_\gamma P^{\gamma\delta})l^{\alpha\beta}\epsilon_{\mu\delta\rho\alpha\beta}\cdot\delta && (81)\\
\{\phi_\rho, \phi'\} &= (\phi+\phi')\partial_\rho\delta - 4(\partial_\lambda P^{\lambda\alpha})D_{\alpha\rho}\cdot\delta && (82)\\
\{\phi, \phi'\} &= (\mathcal{C}^{\rho\sigma}\phi_\rho + \mathcal{C}'^{\rho\sigma}\phi'_\rho)\partial_\sigma\delta && (83)
\end{aligned}$$

where the prime denotes evaluation at a point on the world volume of local coordinates $(\sigma'_1, \cdots, \sigma'_5)$. $\mathcal{C}^{\rho\sigma}$ and $D^{\alpha\rho}$ are local functions of the canonical variables. ϕ_ρ may be realized in terms of the "topological" expression (16) (since it is independent of any metric on the world volume)

$$\phi_\rho = \Pi_a\partial_\rho X^a + \frac{1}{4}V_\rho \tag{84}$$

where

$$V_\rho = \epsilon_{\rho\alpha\beta\gamma\delta} l^{\alpha\beta} l^{\gamma\delta} \tag{85}$$

$$l^{\mu\nu} = \frac{1}{2}(P^{\mu\nu} + {}^\star H^{\mu\nu}) \tag{86}$$

We propose for ϕ the expression

$$\phi = \frac{1}{2}\Pi^a \Pi_a + W \tag{87}$$

$W = gF(y, z)$ where F is to be determined in order to satisfy (83). We notice that (82) is satisfied by any scalar density of weight 1, in the sense $\phi = g \cdot$ (scalar field), since ϕ_ρ given by (84) generates the diffeomorphisms on the spatial 5 dimensional sector of the world volume. The general solution for $F(y, z)$ is given by the space of solutions of the partial differential equation –We refer to Appendix A for the detail calculations of (83)–.

$$2\frac{\partial F}{\partial z} z + \left(\frac{\partial F}{\partial z}\right)^2 z + y\frac{\partial F}{\partial z}\frac{\partial F}{\partial y} + \left(\frac{\partial F}{\partial y}\right)^2 = 2F - 2z\frac{\partial F}{\partial z} - 2y\frac{\partial F}{\partial y}, \quad F \neq 0 \tag{88}$$

In particular, the following is a solution of (88):

$$F = 2y + \lambda\sqrt{1 + y + z} + \frac{\lambda^2}{8} + 2 \tag{89}$$

for any value of λ.

The final form of the constraint for this solution is then given by

$$\phi = \frac{1}{2}\Pi^a \Pi_a + g\left[2y + \lambda\sqrt{1 + y + z} + \frac{\lambda^2}{8} + 2\right] \tag{90}$$

and replacing the expression for y and z we have

$$\phi = \frac{1}{2}\Pi^a \Pi_a + g\left[g^{-1} l_{\alpha\beta} l^{\alpha\beta} + \lambda\sqrt{1 + \frac{1}{2} g^{-1} l_{\alpha\beta} l^{\alpha\beta} + \frac{1}{64} g^{-1} g^{\mu\nu} V_\mu V_\nu} + \frac{\lambda^2}{8} + 2\right] \tag{91}$$

When $\lambda = 0$ we obtain the constraint associated to the Lagrangian (1), which is quadratic on the field $B_{\mu\nu}$. It is interesting to study which could be the Lagrangian associated to the more general constraint (91). It turns out to be the Lagrangian (1) with cosmological term $\frac{\lambda}{4}\sqrt{G}$ added.

Indeed, the canonical analysis of such an action along the lines presented in section (2) yield constraint (91). Explicitly we have

$$L = 2n\sqrt{gM} - \frac{1}{4}N^{\rho}\hat{V}_{\rho} + \frac{1}{2}{}^{\star}H^{\mu\nu}\partial_0 B_{\mu\nu} + \frac{\lambda}{2}ng^{1/2} \tag{92}$$

from which we obtain

$$\Pi_a = 2\frac{g^{1/2}}{n}(-\dot{X}_a + N^{\lambda}\partial_{\lambda}X_a)T - \frac{1}{4}\hat{V}^{\rho}\partial_{\rho}X_a \tag{93}$$

where

$$T = (M^{1/2} + \frac{\lambda}{4}) \tag{94}$$

and from (93) –instead of (8)– we obtain (84) and (91) in the gauge $P^{\mu\nu} = H^{\mu\nu}$.

We notice in (91), that the term $gg^{\mu\nu} = \frac{1}{4!}\epsilon^{\mu\nu_1\nu_2\nu_3\nu_4}g_{\nu_1\lambda_1}g_{\nu_2\lambda_2}g_{\nu_3\lambda_3}g_{\nu_4\lambda_4}\epsilon^{\nu\lambda_1\lambda_2\lambda_3\lambda_4}$, hence it may be rewritten in terms of the covariant metric $g_{\mu\nu}$. The expression (91), consequently, may be expressed in terms of $g_{\mu\nu}$, it is not assumed the existence of the inverse. In order that the scalar z may be involved in an expression with that property, it must appear as

$$z^{\alpha}, \quad \alpha \leq \frac{1}{2} \tag{95}$$

The expression will then be non-polinomic. The most general polinomic solution to (88) which does not depend on the inverse metric is then

$$F = ay + \frac{1}{2}a^2 \tag{96}$$

for any real number $a \neq 0$. However a must be at least $a > 0$ in order to have a bounded from below Hamiltonian. It then may be rescaled to a fix number since the transformations

$$\begin{cases} \Pi \longrightarrow \lambda\pi & l \longrightarrow l, \\ X \longrightarrow \frac{1}{\lambda}X & a \longrightarrow \lambda^6 a, \\ \phi \longrightarrow \lambda^2\phi & . \end{cases} \tag{97}$$

leave the canonical Lagrangian invariant. The property, of the realization of the algebra, for the M5-brane of being only dependent on $g_{\mu\nu}$ and not on its inverse is preserved when we consider a dimensional reduction to 5 dimensional world volume. If we impose the double dimensional reduction, in the sense

$$\begin{aligned} x^5 &= \sigma^5 \\ \frac{\partial}{\partial\sigma^5} &= 0 \end{aligned} \tag{98}$$

we then obtain the following realization of the algebra

$$\{\phi_i, \phi'_j\} = \phi_j \partial_i \delta + \phi'_i \partial_j \delta - (\partial_k P^{kl})^{\star} H^{5m} \epsilon_{ijlm} \delta \tag{99}$$

$$\{\phi_i, \phi'\} = (\phi + \phi')\partial_i \delta - 4(\partial_l P^{lm}) l_{mi} \cdot \delta \tag{100}$$

$$\{\phi, \phi'\} = (\mathcal{C}^{ik}\phi_i + \mathcal{C}'^{ik}\phi'_i)\partial_k \delta \tag{101}$$

where,

$$\phi_k = \Pi_a \partial_k X^a + \frac{1}{2}\epsilon_{mnlk} P^{mn\star} H^l \tag{102}$$

$$\phi = \frac{1}{2}\Pi^2 + 2g + 2(\frac{1}{8}P^{ij}P^{kl}g_{ik}g_{jl} + {}^{\star}H^{i\star}H^j g_{ij}) + \frac{1}{32}(\frac{1}{4}\epsilon_{mnlk}P^{mn}P^{lk})^2 \tag{103}$$

$$\mathcal{C}^{ik} = 4(gg^{ik} + \frac{1}{4}P^{ij}P^{kl}g_{jl} + {}^{\star}H^{i\star}H^k) \tag{104}$$

$${}^{\star}H^i = \frac{1}{6}\epsilon^{ijkl}H_{jkl} \tag{105}$$

This theory incorporates the singular configurations of the metric to the 4-brane theory. It does correspond to it in the flat limit [11] and couples consistently the antisymmetric field with the induced metric. However a direct check showing the equivalence of the canonical Lagrangian, arising from these constraints, and the usual 4-brane one has not be performed. The Hamiltonian constraint is now quartic in the antisymmetric field. We have explicitly checked the closure of this algebra with the structure functions given. It exactly correspond to the dimensional reduction of the 6 dimensional algebra previously obtained.

6 The subspace of solutions with flat induced metric

In this section we shall show how can we recover the master canonical action [11] —with the flat induced metric— starting from the canonical action constructed with the canonical Hamiltonian (12), which is quadratic in the antisymmetric field. To achieve this goal, we will rely on a particular solution of the field equations

Lets consider the field equations arising from the canonical Lagrangian

$$L = \Pi_a \dot{X}^a + P^{\mu\nu} \dot{B}_{\mu\nu} - \mathcal{H} \tag{106}$$

where

$$\mathcal{H} = \Lambda\phi + \Lambda^\alpha \phi_\alpha + \Theta_{5i}\Omega^{5i} + \Theta_j \Omega^j \tag{107}$$

is the canonical Hamiltonian with the general form of the Hamiltonian constraint obtained in section (5).

By taking variations of (106) with respect to Π_a and X^a, Λ and Λ^α we obtain

$$\begin{aligned} \phi &= 0 & (108)\\ \phi_\alpha &= 0 & (109)\\ \dot{X}^a &= \Lambda \Pi^a + \Lambda^\alpha \partial_\alpha X^a & (110)\\ -\dot{\Pi}_a &= \Lambda \frac{\delta\phi}{\delta X^a} - \partial_a [\Lambda^\alpha \Pi_\alpha] & (111) \end{aligned}$$

We now consider the subspace of solutions of the field equations which satisfy initially

$$\begin{aligned} X^0 &= \tau_0 \\ X^\alpha &= \sigma^\alpha \quad \alpha = 1, \cdots, 5 \\ X^a &= 0, \quad a \geq 6 \\ \Pi_a &= 0, \quad a \geq 6 \end{aligned} \tag{112}$$

We impose gauge conditions in the Lagrange multipliers

$$\begin{aligned}\Lambda &= \frac{1}{\Pi^0} \\ \Lambda^\alpha &= -\Lambda\Pi^\alpha,\end{aligned} \tag{113}$$

We then obtain from (108), (109), (110) that (112) are preserved by the evolution equations and $X^0 = \tau$. The explicit expressions for the constraints (108) and (109) , (91) and (16) respectively, provide us the values of Π_0 and Π_α in terms of the antisymmetric field and their conjugate momenta, specifically from (109) we can get the values for Π_α

$$\Pi_\alpha = -\frac{1}{4}V_\alpha \tag{114}$$

and replacing this result in (108) we obtain Π^0

$$-\frac{1}{2}\Pi_0^2 + \frac{1}{32}V_\alpha V^\alpha + g(2y + \frac{\lambda^2}{8} + \lambda M^{1/2} + 2) = 0 \tag{115}$$

$$\Pi_0 = 2\left(M^{1/2} + \frac{\lambda}{4}\right) \tag{116}$$

where $M = 1 + y + z$.
Replacing the expressions for Π_0 and Π_α in the canonical Lagrangian (106) we finally obtain the canonical Lagrangian over a flat induced metric,

$$L = P^{\mu\nu}\dot{B}_{\mu\nu} - 2\left(M^{1/2} + \frac{\lambda}{4}\right) + \Theta_{5i}\Omega^{5i} + \Theta_j\Omega^j \tag{117}$$

When $\lambda = 0$ we recover the master canonical action [11] for the Perry and Schwarz formulation [10].

7 Conclusions

We performed a complete canonical analysis of the bosonic M-theory five brane action corresponding to the partial gauge fixed formulation of the PST action where the scalar field is

fixed as the world volume time. This canonical formulation is quadratic in the dependence on the antisymmetric field and it has second class constraints. We removed the second class constraints by proposing an extension of the canonical action derived from the covariant action of [1] and constructed a *master canonical action* with first class constraints only, preserving the locality of the field theory. We then constructed the associated nilpotent BRST charge, assuming a world volume with a compact without boundary spatial part, and its BRST invariant effective theory. The BRST charge is well defined even for configurations in which the induced metric has zero determinant at some point or open neighborhood of the world volume. It does not require the existence of the inverse of the induced metric. Consequently, the BRST effective action, which is manifestly Lorentz invariant in the target space and manifestly BRST invariant in the world volume, has also the same property. The singular configurations have then to be considered as physical one. This implies the existence of configurations changing the topology of the M5-brane without changing its energy. We expect then that they will have the same consequences as for the $D = 11$ supermembrane with respect to its spectrum once the supersymmetry is implemented into the theory.

We obtained the physical Hamiltonian of the theory in the LCG from the general effective action (55) and analyzed its stability properties explicitly. We showed the existence of singular configurations, where $g = 0$, even for maximum rank of the antisymmetric field. We also constructed global configurations where singularities are not allowed at any point of the world volume, and give a geometrical interpretation of them in terms of higher order bundles.

Finally, by studying the algebra of 6 dimensional diffeomorphisms we found the most general structure for the Hamiltonian constraint and we identified the constraint associated with the bosonic five brane action upgraded with a cosmological term as a constraint with a Born-Infeld type term. The M5-brane may be characterized from this algebraic point of view as been the only one whose Hamiltonian constraint is polinomial in the antisymmetric field and is well defined without assuming the existence of the inverse of the induced metric.

We have now all elements to start analyzing the spectrum of the super M5-brane. In particular to look at the massless states of the theory. It will be important to relate this problem to its dual one for the $D = 11$ super membrane, where the spectrum was determined only for flat Minkowski target space using an $SU(N)$ regularization. Another aspect that may follow from our results is the explicit construction of the Seiberg-Witten map relating the M5-brane to a

non-commutative geometries in terms of the BRST cohomology. In this sense, the simplectic structure we have constructed may be important.

Acknowledgements

We would like to thank I. Bandos, M. Caicedo, L. Quevedo, D. Sorokin and J. Stephany for interesting discussions. We also would like to thank the theoretical group at Department of Mathematics, Kings College for kind hospitality and the Research Deanery, Grant G11, at USB for financial support.

A Appendix

We will evaluate (15) in terms of the functions $F(y,z)$.

We have

$$\{\phi,\phi'\} = \{\frac{1}{2}\pi^a\pi_a + gF, \frac{1}{2}\pi'^a\pi'_a + g'F'\} \tag{118}$$

which by explicit evaluation yields

$$\begin{aligned}
\{\phi,\phi'\} &= \pi^a\left[\{\pi_a, g'\}F' + g'\frac{\partial F'}{\partial z'}\{\pi_a, z'\} + g'\frac{\partial F'}{\partial y'}\{\pi_a, y'\}\right] \\
&+ g\frac{\partial F}{\partial z}\{z,z'\}g'\frac{\partial F'}{\partial z'} + g\frac{\partial F}{\partial z}\{z,y'\}g'\frac{\partial F'}{\partial y'} + g\frac{\partial F}{\partial y}\{y,z'\}g'\frac{\partial F'}{\partial z'} \\
&+ g\frac{\partial F}{\partial y}\{y,y'\}g'\frac{\partial F'}{\partial y'}
\end{aligned} \tag{119}$$

The Piosson brackets of $P^{\mu\nu}$ and ${}^\star H^{\mu\nu}$ is given by

$$\begin{aligned}
\{P^{\mu\nu}, {}^\star H'^{\hat{\mu}\hat{\nu}}\} &= \frac{1}{2}\epsilon^{\hat{\mu}\hat{\nu}\hat{\rho}\hat{\sigma}\hat{\lambda}}\{P^{\mu\nu}, \partial'_{\hat{\rho}}B'_{\hat{\lambda}\hat{\sigma}}\} \\
&= -\epsilon^{\hat{\mu}\hat{\nu}\hat{\rho}\mu\nu}\partial'_{\hat{\rho}}\delta(\sigma'-\sigma)
\end{aligned} \tag{120}$$

we then obtain

$$\begin{aligned}
\{l^{\mu\nu}, l'^{\hat{\mu}\hat{\nu}}\} &= \frac{1}{4}\left[\{P^{\mu\nu}, {}^\star H'^{\hat{\mu}\hat{\nu}}\} + \{{}^\star H^{\mu\nu}, P'^{\hat{\mu}\hat{\nu}}\}\right] \\
&= \frac{1}{2}\epsilon^{\hat{\mu}\hat{\nu}\hat{\rho}\mu\nu}\partial_{\hat{\rho}}\delta
\end{aligned} \tag{121}$$

We also get

$$\begin{aligned}
gg'[\{z,y'\} + \{y,z'\}] &= \{\frac{1}{2}g_{\mu\alpha}g_{\nu\beta}l^{\mu\nu}l^{\alpha\beta}, \frac{1}{64}g'^{\hat{\mu}\hat{\nu}}V_{\prime\hat{\mu}}V_{\prime\hat{\nu}}\} + \{\frac{1}{64}g^{\mu\nu}V_\mu V_\nu, \frac{1}{2}g'_{\hat{\mu}\hat{\alpha}}g'_{\hat{\nu}\hat{\beta}}l'^{\hat{\mu}\hat{\nu}}l'^{\hat{\alpha}\hat{\beta}}\} \\
&= \frac{1}{32}g_{\mu\alpha}g_{\nu\beta}l^{\mu\nu}V'^{\hat{\mu}}\{l^{\alpha\beta}, V'_{\hat{\mu}}\} + \frac{1}{32}g'_{\mu\alpha}g'_{\nu\beta}l'^{\mu\nu}V^{\hat{\mu}}\{V_{\hat{\mu}}, l'^{\alpha\beta}\} \\
&= \frac{1}{32}\left[(l_{\alpha\beta}l^{\hat{\alpha}\hat{\beta}}V^{\hat{\mu}}\epsilon_{\hat{\mu}\hat{\alpha}\hat{\beta}\hat{\sigma}\hat{\lambda}}) + ()'\right]\epsilon^{\alpha\beta\hat{\sigma}\hat{\lambda}\gamma}\partial_\gamma\delta
\end{aligned} \tag{122}$$

and

$$
\begin{aligned}
gg'\{z,z'\} &= \frac{1}{64}\frac{4}{64}V^{\nu}V'^{\hat{\nu}}\{V_{\nu},V'_{\hat{\nu}}\} \\
\{V_{\nu},V'_{\hat{\nu}}\} &= 4\epsilon_{\alpha\beta\nu\sigma\lambda}\epsilon_{\hat{\alpha}\hat{\beta}\hat{\nu}\hat{\sigma}\hat{\lambda}}l^{\sigma\lambda}l'^{\hat{\sigma}\hat{\lambda}}\{l^{\alpha\beta},l'^{\hat{\alpha}\hat{\beta}}\} \\
&= 4\epsilon_{\alpha\beta\nu\sigma\lambda}\epsilon_{\hat{\alpha}\hat{\beta}\hat{\nu}\hat{\sigma}\hat{\lambda}}l^{\sigma\lambda}l'^{\hat{\sigma}\hat{\lambda}}\frac{1}{2}\epsilon^{\alpha\beta\hat{\alpha}\hat{\beta}\gamma}\partial_{\gamma}\delta \\
gg'\{z,z'\} &= \frac{1}{4}\frac{1}{16}\frac{1}{16}\left[(\epsilon_{\alpha\beta\nu\sigma\lambda}V^{\nu}l^{\sigma\lambda})(\epsilon_{\hat{\alpha}\hat{\beta}\hat{\nu}\hat{\sigma}\hat{\lambda}}V^{\hat{\nu}}l^{\hat{\sigma}\hat{\lambda}})+()'_{\alpha\beta}()'_{\hat{\alpha}\hat{\beta}}\right]\epsilon^{\alpha\beta\hat{\alpha}\hat{\beta}\gamma}\partial_{\gamma}\delta
\end{aligned}
\tag{123}
$$

We finally obtain

$$
\{W,W'\} = I + II + III \tag{124}
$$

where

$$
\begin{aligned}
\mathrm{I} &= \left[\frac{\partial F}{\partial y}\frac{\partial F}{\partial y}\frac{1}{4}gV_{\sigma}g^{\sigma\lambda}+'\right]\partial_{\lambda}\delta \\
\mathrm{II} &= \left[\frac{\partial F}{\partial z}\frac{\partial F}{\partial y}\frac{1}{4}\left(l_{\mu\alpha}l^{\alpha\lambda}V^{\mu}+\frac{1}{2}l_{\alpha\beta}l^{\alpha\beta}V^{\lambda}\right)+'\right]\partial_{\lambda}\delta \\
\mathrm{III} &= \left[\frac{\partial F}{\partial z}\frac{\partial F}{\partial z}\frac{1}{64}\frac{1}{4}V^{\lambda}V_{\mu}V^{\mu}+'\right]\partial_{\lambda}\delta
\end{aligned}
\tag{125}
$$

We will now evaluated Poisson brackets related to X^a and Π_a.
We get

$$
\{\pi^a, g'_{\alpha\beta}\} = -2\partial'_{\alpha}\delta(\sigma'-\sigma)\partial'_{\beta}x'^a \tag{126}
$$

which may be used to evaluate

$$
\begin{aligned}
\frac{1}{2}\{\pi^a\pi_a, W'\} &= \pi^a\left[\{\pi_a,g'\}F'+\{\pi_a,F'\}g'\right] \\
&= \pi^a\left[\{\pi_a,g'\}F'+g'\frac{\partial F'}{\partial z'}\{\pi_a,z'\}+g'\frac{\partial F'}{\partial y'}\{\pi_a,y'\}\right]
\end{aligned}
\tag{127}
$$

we obtain after some calculations

$$\frac{1}{2}\{\pi^a\pi_a, W'\} = \mathrm{A} + \mathrm{B} + \mathrm{C} \tag{128}$$

with

$$\begin{aligned}
\mathrm{A} &= \left[2Fgg^{\sigma\lambda}\pi^a\partial_\sigma x_a + \quad \prime \quad\right]\partial_\lambda\delta \\
\mathrm{B} &= \left[\frac{\partial F}{\partial y}g^{\sigma\lambda}l_{\alpha\beta}l^{\alpha\beta}\pi^a\partial_\sigma x_a + 2\frac{\partial F}{\partial y}g_{\mu\alpha}l^{\mu\sigma}l^{\alpha\lambda}\pi^a\partial_\sigma x_a + \quad \prime \quad\right]\partial_\lambda\delta \\
\mathrm{C} &= -\frac{1}{32}\left[\frac{\partial F}{\partial z}g^{\sigma\lambda}g^{\mu\nu}V_\mu V_\nu\pi^a\partial_\sigma x_a + \frac{\partial F}{\partial z}g^{\mu\sigma}g^{\nu\lambda}V_\mu V_\nu\pi^a\partial_\sigma x_a + \quad \prime \quad\right]\partial_\lambda\delta
\end{aligned} \tag{129}$$

We now use (125) y (129) to obtain the closure of the algebra of constraints, that is to restrict $F(y,z)$ in order to satisfy (15). After some calculations we obtain the following partial differential equation to be satisfied by F,

$$2\frac{\partial F}{\partial z}z + \left(\frac{\partial F}{\partial z}\right)^2 z + y\frac{\partial F}{\partial z}\frac{\partial F}{\partial y} + \left(\frac{\partial F}{\partial y}\right)^2 = 2F - 2z\frac{\partial F}{\partial z} - 2y\frac{\partial F}{\partial y} \tag{130}$$

References

[1] Paolo Pasti, Dmitri Sorokin, and Mario Tonin. *Phys. Lett.*, B398:41, 1997.

[2] Igor Bandos, Kurt Lechner, Alexei Nurmagambetov, Paolo Pasti, Dmitri Sorokin, and Mario Tonin. *Phys.Rev. Lett.*, 78:4332, 1997.

[3] P.S. Howe and E. Sezgin. *Phys. Lett.*, B 394:62, 1997.

[4] P.S. Howe and E. Sezgin and P. C. West. *Phys. Lett.*, B 399:49, 1997.

[5] O. Barwald, N. Lambert, and P. C. West. On the energy momentum tensor of the m-theory fivebrane. hep-th/9904097.

[6] M. Aganagic, J. Park, C. Popescu, and J. Schwarz. *Nucl. Phys. B*, 496:191, 1997.

[7] Bergshoeff and P. K. Townsend. unpublished.

[8] J. P. Gauntlett, Joaquin Gomis, and P.K. Townsend. *JHEP*, 01:3, 1998.

[9] E. Bergshoeff, D. Sorokin, and P. K. Townsend. *Nucl. Phys.*, B533:303, 1998.

[10] M. J. Perry and J. H. Schwarz. *Nucl. Phys. Lett. B*, 498:47, 1997.

[11] A. De Castro and A. Restuccia. *Class. Quantum Grav.*, 16:1, 1999.

[12] N. Seiberg and E. Witten. *Nucl. Phys.*, B426:19–52, 1994. hep-th/9407087.

[13] M. Caicedo, I. Martín, and A. Restuccia. *Proc. I SILAFAE, Yucatán, México*, page 459, 1997. hep-th/9701010.

[14] R. Gianvittorio, A. Restuccia, and J. Stephany. *Mod. Phys. Lett. A*, 6(23):2121, 1991.

[15] B. de Wit, J. Hoppe, and H. Nicolai. *Nucl. Phys.*, B305:545, 1988.

[16] B. de Wit, M. Lusher, and H. Nicolai. *Nucl. Phys.*, B320:135, 1989.

[17] Hermann Nicolai and Robert Helling. Supermembranes and Matrix Theory. hep-th/9809103.

[18] Marc Henneaux. *Phys. Rep.*, 12C, 1985.

[19] I. Batalin and E. Fradkin. *Phys. Lett.*, B 122:157, 1983.

[20] M. Cicedo and A Restuccia. *Class. Quantum Grav.*, 10:833, 1993. ibid *Phys. Lett.* B307:77, 1993.

[21] C. Teitelboim. *Phys. Rev.*, D25:3159, 1982.

[22] I. Martín, A. Restuccia, and R. Torrealba. *Nucl. Phys.*, B 521:117, 1998.